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ISBN: 978-1-890160-77-7

STEAM EDUCATION

An Interdisciplinary Look at Art in the Curriculum

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NATIONAL ART EDUCATION ASSOCIATION

Connecting Mathematics to Verbal–Visual Art

Punya Mishra • Danah Henriksen

> ***The desire for symmetry, for balance, for rhythm in form . . . is one of the most inveterate of human instincts.***
>
> —Edith Wharton, *Complete Works of Edith Wharton*

> ***By all means break the rules, and break them beautifully, deliberately and well.***
>
> —Robert Bringhurst, *The Elements of Typographic Style*

Distorting visual letterforms and writing words to express mathematical symmetries may appear to be a strange way of engaging with math learning. But this is precisely the type of creative approach that high school students applied at a local school in Phoenix during a creative math–art lesson with the first author. Working individually or collectively, students grappled with the task of exploring the perceptual flexibility of shape perception and pattern formation through creative visual wordplay.

In this piece, we will discuss the use of "ambigrams"—a neologism created by the cognitive scientist Douglas Hofstadter (Polster, 2000)—for exploring and learning about mathematical symmetry. Ambigrams are a form of visual wordplay focused on writing words as an art form that allows them to be read in multiple ways. We situate this in a real-world math lesson done with high school students of multiple ages, where they created new and surprising ways of writing words such that they could be read when rotated 180 degrees, or when reflected in a mirror or as a fractal. These activities required learners to see words both as carriers of meaning *and* as geometric shapes that can be artfully manipulated and played with. For the students, the process was at times frustrating and challenging yet fun, and it connected visual artistry with mathematical concepts for a unique, creative learning activity.

Creativity is a driving force in mathematics. As Manjul Bhargava, Fields Medal winner in mathematics, wrote:

> For mathematicians, mathematics—like music, poetry, or painting—is a creative art. All these arts involve—and indeed require—a certain creative fire. They all strive to express truths that cannot be expressed in ordinary everyday language. And they all strive towards beauty (as cited in Rajghatta, 2014, para. 26).

It can be challenging, however, to find approaches that blend math and visual arts to help students recognize the inherent creative connection between the disciplines. Research demonstrates that creativity is not "magical"; instead, creative ideas emerge from combining or recombining preexisting ideas and concepts in unique and new ways (Ferguson, 2011). We suggest that math lessons can draw upon three principles of artistic creativity to help teachers integrate mathematics and the arts in science, technology, engineering, the arts, and math (STEAM) approaches.

The ambigram lesson we will share later is grounded in these creative principles: (1) learning to see, (2) creativity through constraints, and (3) the importance of play. Despite our specific focus on ambigram lessons, these can be useful to teachers in designing any STEAM-based lesson. Therefore, we begin with an overview of the principles before discussing how they played out in the ambigram lesson.

Creative Foundations for STEAM Lessons

Each of the principles used in this lesson, of learning to see, creativity through constraints, and the importance of play, are central to the work of creativity across disciplines. *Learning to see* is important because in creative practices (whether in math, art, or any discipline) students must learn to see the world to notice details and identify existing patterns—and then go beyond that to imagine new and interesting possibilities. Root-Bernstein and Root-Bernstein (2013) describe this ability to observe the world and identify existing patterns (then to create new ones) as a big-picture thinking skill for creativity across disciplines. This skill is foregrounded in the visual arts. Visual artists experience the world through observant fascination. Christensen (2019) notes that a painter might be enchanted by the color of a shadow, or marvel at shapes in a flower's petal. Trained artists view seemingly mundane sights as compelling. Rather than seeing an apple as an apple, they might scrutinize its form and shifting shades of color, or examine the curving shape, texture, and the play of light and dark or the shape of the shadows. By seeing what is actually there, rather than what the mind assumes is there, an artist learns to truly see (Thomas et al., 2008), which allows for a new creation. Anyone can develop this skill.

Importantly, *creativity occurs through constraints*. While creative people are open-minded, creativity is best expressed by bringing imagination to the constraints of the task at hand. Without constraints, a project or task becomes too open and potentially chaotic to produce something new and effective (Onarheim, 2012). Research reveals that creative work benefits from *appropriate* proportions of constraint to offer some guides or expectations for the task at hand (Rosso, 2014). The meaning of "appropriate" varies—but in general it is best to avoid task designs that are so open they are unlimited and undirected, but also not be so constrained as to impede fresh possibilities and new ideas. Discovering or inventing solutions within constraints is crucial to creativity.

Finally, the *importance of play* in creativity is essential. "Play" is often thought of as a childhood activity—something for fun, like games or sports. This just-for-fun view of play sometimes precludes it from classroom learning (i.e., it is seen as being for recess or outside of lessons). But play is critical in the creative process for people of any age (Conklin, 2014). Play brings joy, fun, and meaning and is foundational to how students learn and develop creatively. Mishra et al. (2011) state that "deep" or transformational learning play is open-ended—it involves toying with ideas to inspire creative, boundary-breaking thinking. It is a cognitive skill that develops whenever students get to play with ideas, signs, symbols, or artifacts in an open-ended way, to see what comes of it. Play is essential to creative learning, letting students extend their thinking and trial ideas.

STEAM education involves developing learning approaches that cut across disciplinary lines, and we share these three principles as a frame to structure lessons using mathematics and art, which are part of the ambigram examples, below. These ideas are also applicable and transferable to other types of STEAM lessons that teachers might create.

Working With Ambigrams

Ambigrams are a way of writing words (in artistic rendering) such that they can be read or interpreted multiple ways. Ambigrams exploit *how* words are written. In doing so, they bring together the mathematics of symmetry, the elegance of typography, *and* the psychology of visual perception to create surprising, artistic designs.

Figures 1 and 2 provide examples of words written such that they can be read the same when rotated 180 degrees.

Figure 1. Ambigram for "wordplay"—invariant under rotation (i.e., flip it 180 degrees vertically or horizontally and it reads the same).

Figure 2. Ambigram for "ambigram"—invariant under rotation (i.e., flip it 180 degrees vertically or horizontally and it reads the same).

Ambigrams come in many styles. Consider Figure 3, where the word "right," when reflected across the 45-degree axis, reads "angle." Ambigrams can also come in chains, such as this design for the word "infinity," mapped into the infinity symbol (Figure 4). Ambigrams can also be similar to tessellations (akin to the graphic art of M.C. Escher). See, for instance, the design for the word "space" (Figure 5), where replications of the word map on to the surface of a sphere.

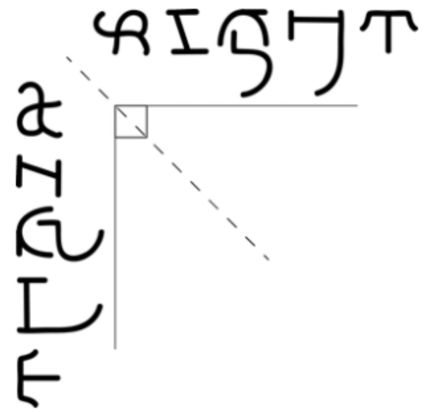

Figure 3. The word "right" becomes "angle" when reflected 45 degrees. Inspired by a solution first put forth by Bryce Herdt.

Figure 4. The word "infinity" written such that it can be read when rotated, and also makes the symbol for infinity.

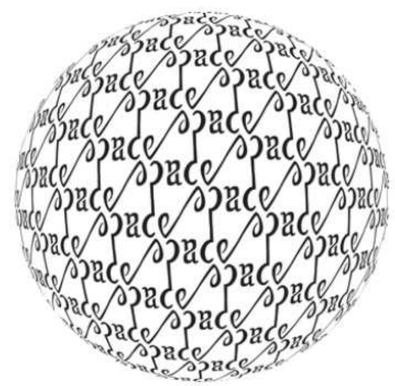

Figure 5. A space-filling design for the word "space"—a verbal–visual tessellation similar to M.C. Escher's tessellations.

At heart, ambigrams are about symmetry, a critical concept in geometry, science, and art. In this context, ambigrams can speak to specific mathematical transformations and relationships. For instance, mathematicians speak of four specific transformations we can use to manipulate a geometrical shape: rotation (Figure 6), reflection (Figures 7 and 8), translation (Figure 9), and dilation/resizing (Figure 10).

Figure 6. A 180-degree rotational ambigram for the word "rotate."

Figure 7. A vertical-reflection ambigram for the word "algebra," symmetric around the vertical axis.

Figure 8. A "lake" reflection for the world "Chicago," symmetric around the horizontal axis.

Figure 9. A set of shapes that reads the word "translate" when moved horizontally.

Figure 10. A dilation ambigram for the word "Zoom," which reads the same when resized.

Mathematicians are not satisfied with just these four moves. For instance, they like to combine two or more transformations (such as a reflection *followed by* a translation or rotation) on the same object. For instance, see the rich design for the word "fractal" (Figure 11). A fractal is a geometric pattern that recursively repeats itself at ever-smaller (or ever-larger) scales. Similarly, here is a design for the "sine curve" (Figure 12) showing both rotational and translation symmetry (a property of the mathematical idea of the sine curve).

Figure 11. Fractal ambigram for the word "fractal," which reads the same when you rotate and zoom into or away from the design.

These examples situate and ground the abstractions of mathematics within the words and are a form of artistic wordplay. These explorations allow learners to grasp, in an aesthetic and fun manner, ideas related to symmetry, paradox, limits, and infinity. The act of appreciating ambigrams and creating them could allow learners to see these deeper patterns and to experience these ideas.

The process of creating ambigrams connects deeply to the three themes we described above. Taking each in turn:

Figure 12. A design for the word "sine" that captures the rotational, translational symmetry of the sine curve.

1. *Learning to see*: We can all read, but we often do not "see" letterforms and shapes. Creating ambigrams requires a dual sensitivity, to see letters and words as both symbolic conveyers of meaning and as complex geometric shapes that can be manipulated. Considering the "right-angle" reflection ambigram, creating it requires one to see how the letter "a," *if written the right way*, could be reflected to the letter "r"—something that is not obvious at first glance.

2. *Creativity through constraints*: There are tight constraints to creating ambigrams. For instance, to create a reflection design (such as the one for "reflect") requires distorting letterforms so they can be symmetric on reflection and remain legible. Essentially, every ambigram has a tight walk between legibility and conveying multiple readings.

3. *The importance of play*: The only way to create ambigrams is to doodle and play with words and letters, and most importantly their shapes. Solutions are not preordained but have to be discovered but sketching out possibilities and just being open to possibilities that emerge.

Underneath the fun exterior, there are deeper discussions on mathematics, language, and human perception. To create ambigrams, one must *play* with words and letters as carriers of meaning but also as abstract shapes that can be manipulated. The act of creating ambigrams pushes the creator to attempt to distort the visual shape of a letterform even while maintaining its "essence," as it were. There is intentionality and serendipity—where solutions can be both surprising and elegant. Douglas Hofstadter, the cognitive scientist, described creating ambigrams as being a "microworld" for the creative process.

Exploring Ambigrams and Mathematics

The first author taught a series of ambigram activities created for students to become creative and flexible in connecting the mathematics and aesthetics of words. Although these activities were done with students in upper middle and high school, variations of these tasks could be given to younger and older learners as well. These are scaffolded exercises that allow students to (1) learn to see the world of letterforms differently, (2) help them understand the constraints inherent in the "game" and (3) provide them structured opportunities to play with shapes and meaning.

The goal for a teacher is to help students learn to play with ideas of symmetry, representation, visual manipulation, and artistic design. Students must be scaffolded into the process. It is not easy to see letterforms as perceptual shapes that can be visually manipulated to convey meaning. Thus, providing examples of ambigrams is important. There are some books (Kim, 1996; Langdon, 2005; Polster, 2008; Prokhorov, 2013) and a range of online resources available that provide examples to inspire students.

Ultimately, there is no shortcut to jumping in and playing with writing words in interesting ways. Our experience has shown that these are deeply engaging activities—and students will jump in immediately. The tasks appear to be intrinsically motivating as students start playing with words and shapes, tweaking them, rotating their writing pads (in which they have been sketching) to see how the words look and whether they are legible. There are also activities and games that can help scaffold students to see letterforms in new ways, as making meaning from geometrical shapes that can be played with in creative ways. Incorporating some collective or individual discussion can be helpful for making connections between students' artistic choices and geometric concepts—strengthening both areas of knowledge.

Figure 13. The first image above is a scanned image of a rotational ambigram for "Jason," created by a student. Below is a cleaned-up version of the same design. The second image highlights the rotational symmetry that the student noticed and manipulated.

Here, we showcase a few student examples and highlight the mathematical properties of their solutions. With just a few minutes of open exploration, students can often come up with some original designs and in certain cases exploit the mathematical dimensions of this art form. Perhaps unsurprisingly, the students often started by writing out their own names, playing with the letters in their names, tweaking shapes to seek patterns and symmetries—to identify visually interesting solutions that work with basic geometrical transformations.

The first few examples, below, focus on constructing rotational symmetric designs of their names. In each, the students tried to find ways to capture half of their name in such a way that when rotated it would make the other half. For instance, the eponymous design for "Jason" can essentially be seen as being constituted in two parts—each a rotational identical of the other, as shown in Figure 13.

The next two examples are similar (and subtly different). Figure 14 shows a design by a student for his name—"Andrew." It has the same rotational symmetry as "Jason" above, with an additional insight involving combining letters to create a compound shape that reads one letter one way and two letters the other way. For instance, consider the manner in which the "An" maps onto "w." The previous example ("Jason,") on the other hand, had a one-to-one mapping of better letters.

Figure 14. The first image is a scan of a rotational ambigram for "Andrew" as sketched by a student. The image below has been cleaned up and colored to highlight (1) the rotational symmetry of the design and (2) the manner in which TWO different letter shapes map into a single letter shape (the "a-n" mapping into "w" in this example).

In the next eponymous design, "Meredith" (Figure 15), the student took this idea of collapsing multiple letters to create one letter to another extreme. In this design, the letter "M" maps onto not one, or two, but rather to *three* different letters, i.e., "M" maps onto "i," "t," and "h." This is a sophisticated way of looking and thinking about letterforms and symmetry, especially for a high schooler who had never heard of this particular art form before.

Figure 15. Scanned image of a rotationally symmetric design for "Meredith" as sketched by a student. Below is the cleaned-up version of the design, highlighting both the rotational symmetry of the design and the manner in which THREE different letters map into a single letter shape (the "m" mapping onto "ith").

The next two designs demonstrate just how creative even beginners in the art of ambigrams can be, with minimal introduction. Figure 16 shows an attempt by a student to create a 90-degree rotational design that reads "fire" one way and "water" the other. Though not entirely successful, the attempt itself demonstrates a willingness to experiment and take creative risk, to see letters both as shapes and carriers of meaning, and to perceive rotational symmetries in the letterforms.

Figure 16. A 90-degree rotational design created by a student where the word "fire" maps onto the word "water," and a cleaned-up version of the same.

Figure 17. Scanned image of a pentagonal design for the name "Alexa," where the student took advantage of the manipulating the letter "l" so that it would work as an "x" when rotated 72 degrees. The lines she drew demonstrate that she thought not just of the words but also of the underlying mathematical symmetries. This can be more clearly seen in the cleaned-up version of the image.

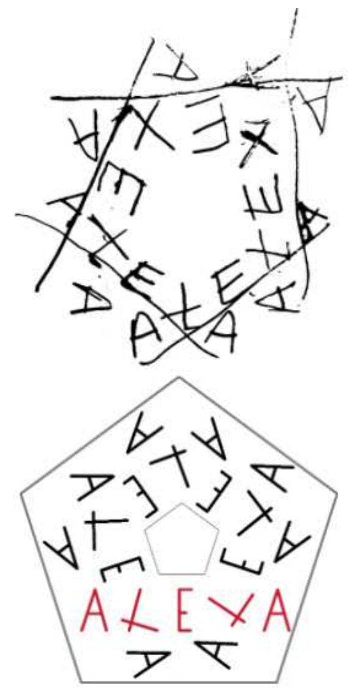

Figure 17 is another geometrically ambitious attempt by a student to create an eponymous design for her name, "Alexa." The student manipulated the letters "l" and "x" so "l" could be read as "x" and vice versa when rotated by 72 degrees. And that is what gives the design its fivefold (pentagonal) symmetry!

How Do Teachers Introduce These Ideas to Students?

An important part of getting students to create ambigrams is to provide them with concrete examples, either from books or websites. Ambigrams are hard to describe but easy to grasp visually. That said, there are activities that can scaffold students' experiments with ambigrams. We provide some examples of these activities below, as well as some insights into how teachers can create their own activities. We suggest the following ambigram activities, lessons, or play can be used alongside discussion to help students make connections between what they create and math concepts.

The activities start with relatively easy problems that push students to see letter and number shapes in new ways and gradually become more complex. These activities then prompt them to manipulate letterforms and shapes to create coherent designs. Thus, the first few activities appear simple and almost as visual puzzles—but they hide deeper insights and ways of looking at letterforms.

Digital Conversion

Figure 18 demonstrates an activity where the students have to see numbers as letter shapes. They have to use all the numbers (0–9) in the top row to spell the names of all the numbers in the boxes below. Basic geometric transformations (rotation and reflection) are allowed. The first example has been completed for them—where the word "zero" has been created by rotating or reflecting the numbers 2, 3, 7 and 0. This activity prepares them for seeing numbers as being shapes that can be "seen" in new ways.

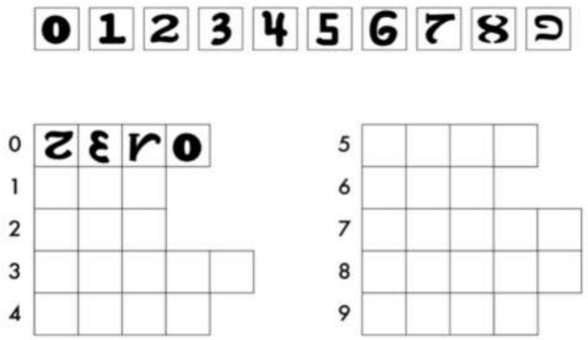

Figure 18. The digital conversion activity (inspired by Scott Kim) is a structured activity that encourages looking at numbers and letterforms as shapes that can take on multiple meanings depending on how they are perceived.

Half-a-Bet. The activity shown in Figure 19 is similar to the previous one, but here the students are asked to rotate and reflect the 13 letters provided to create the rest of the alphabet. The constraint is that you can use each shape just once. This activity allows students, within constraints, to see shapes in new and meaningful ways.

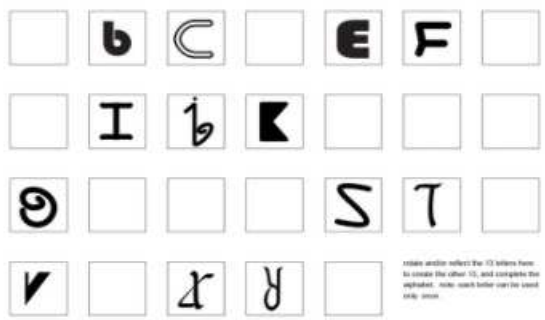

Figure 19. The half-a-bet activity (inspired by Scott Kim) undermines an essentialist reading of the alphabet and encourages constrained yet flexible interpretations of their shapes that can take on alternative meanings depending on how they are perceived.

Split-Personalities I and II. In Figure 20, the goal is to fill in the box with a letter written to have two different meanings, depending on context (e.g., the shape at the center can be seen as both "B" and "13."). The activity that follows goes further, except that it requires rotation by 180 degrees. Again, the goal is to have students drawing and playing with shapes to see how they change meaning in context.

In Conclusion: Aesthetics, Ambigrams, and Mathematics

One of the challenges of mathematics as taught in schools today is how dryly it is approached, and how often it feels divorced from artistic, visual, or captivating abstractions that geometry, puzzles, and problem solving allow for. The pleasure and beauty of playing with ideas, visuals, representations, and symmetries sometimes gets lost in dry curricula, where students do not get to creatively play in hands-on ways. As Bertrand Russell said, speaking of art and mathematics, the "true spirit of delight" can be found in both mathematics and poetry (Russell, 1961/2009, p. 229). Figure 21 captures this idea; that is, "math" becomes "poetry" (and vice versa) when rotated.

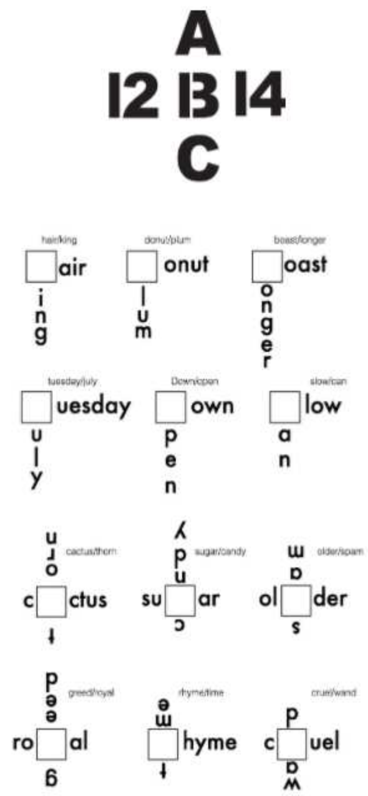

Figure 20. Split-Personalities I and II. These two activities are stepping stones to creating ambigrams. Solving these open-ended visual puzzles requires looking at the shapes of alphabets as being flexible and allowing for multiple readings, depending on context.

The creation of ambigrams can be a highly engaging activity that can lead to seemingly inevitable and yet surprising and elegant solutions, solutions that either take advantage of inherent asymmetries or reveal hidden symmetries. A big part of the learning can come in opportunities to "debrief" with students, getting them to reflect (no pun intended) on their thinking. So, after engaging in these or other ambigram activities, it helps to have a guided discussion where the teacher asks students to share their work and thinking, to connect to deeper ideas of symmetry and meaning-making.

Both mathematicians and artists engage in creative, open-ended play with ideas through manipulating abstract symbols. Ambigrams allow learners across the board to genuinely experience some of these pleasures that playing with ideas can bring and through that develop a deeper appreciation of both mathematics and art.

Figure 21. Rotational ambigram that reads "math" one way, and "poetry" when rotated by 180 degrees.

Author's Note: All designs in this article have been created by Punya Mishra. ©Punya Mishra unless indicated otherwise.

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